Name:

Date: Period:

Circumcenters and Incenters

Lesson Objective IWBAT find and use circumcenters. and incenters of Δs.

Concurrent Lines: When 3 or more lines, rays, or segments intersect in the same point, they are called concurrent lines.

Point of Concurrency: The point of intersection of the lines, rays, or segments.

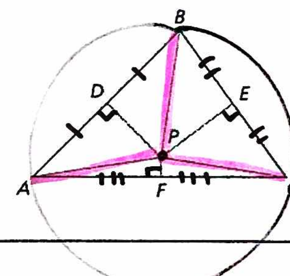
In a triangle, the three ⊥ bisectors of the sides are concurrent.

➔ The point of concurrency is called the circumcenter } definition

Circumcenter Theorem

The circumcenter of a triangle is equidistant from the vertices of the triangle.

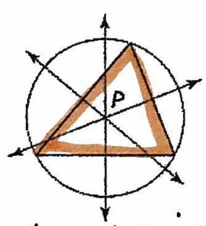
*P is circumcenter



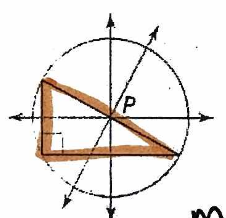
**Remember, the "equidistant" is because this length (AP, CP, and BP in the diagram above) represents the radius of the circle!

**The location of the circumcenter (point P in the diagrams below) depends on the type of triangle.

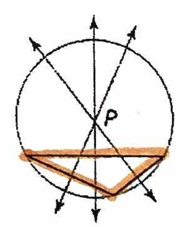
**The circle with center P is said to be circumscribed about the triangle.



acute Δ: inside



right Δ: mdpt of hyp.

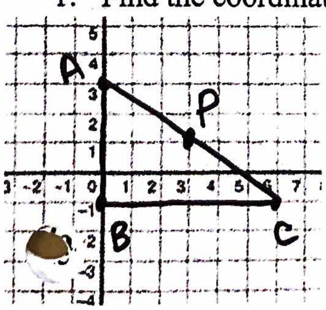


obtuse Δ: outside

Finding the Circumcenter of a Triangle in the Coordinate Plane

Example:

1. Find the coordinates of the circumcenter of ΔABC with vertices A(0, 3), B(0, -1), and C(6, -1).



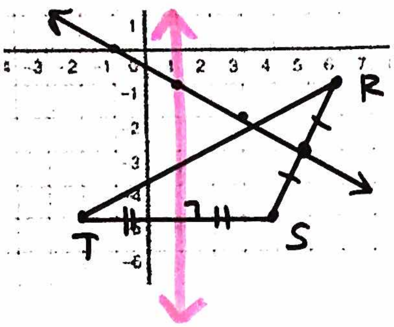
right Δ: mdpt of hyp ($\overline{AC}$)

$\left(\frac{0+6}{2}, \frac{3+(-1)}{2}\right) = \boxed{(3, 1)}$

Think About It... Can you find the coordinates of the circumcenter of a right triangle WITHOUT having to find the ⊥ bisectors?

Example:

2. Find the coordinates of the circumcenter of ΔRST with vertices R(6, -1), S(4, -5) and T(-2, -5).



$\perp$ bisector of $\overline{RS}$

① mdpt $(5,-3)$

② slope $= 2$

③ $\perp$ slope $= -\frac{1}{2}$

$y+3=-\frac{1}{2}(x-5)$

$2\left(y+3\right)=\left(-\frac{1}{2}x+\frac{5}{2}\right)2$

$2y+\cancel{6}=-x+5$

$x+2y=-1$

$x=1$

solve system

$1+2y=-1$

$2y=-2$

$y=-1$

$\boxed{(1,-1)}$

Just as a triangle has three $\perp$ bisectors, it also has three angle bisectors.

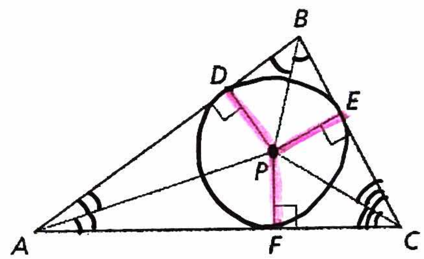
The angle bisectors of a triangle are also concurrent. This point of concurrency is the incenter of the triangle.

*definition

Incenter Theorem

The incenter of a triangle is equidistant from the sides of the triangle.

*P is incenter



**Remember, the "equidistant" is because this length (DP, EP, and FP in the diagram above) represents the radius of the circle!

Examples:

3. In the figure shown, $ND = 5x - 1$ and $NE = 2x + 11$.

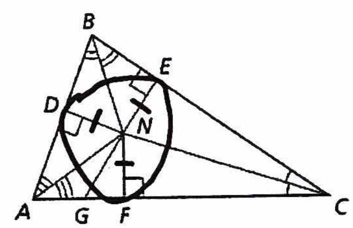
   a. Find NF.

   $5x-1=2x+11$

   $x=4$

   $NF=DN$

   $\boxed{NF=19}$



   b. Can NG be equal to 18? Explain your reasoning.

   NF is shortest dist. from N to $\overline{AC}$. So NG is >19. cannot be 18.

   *N is incenter equidist. from sides

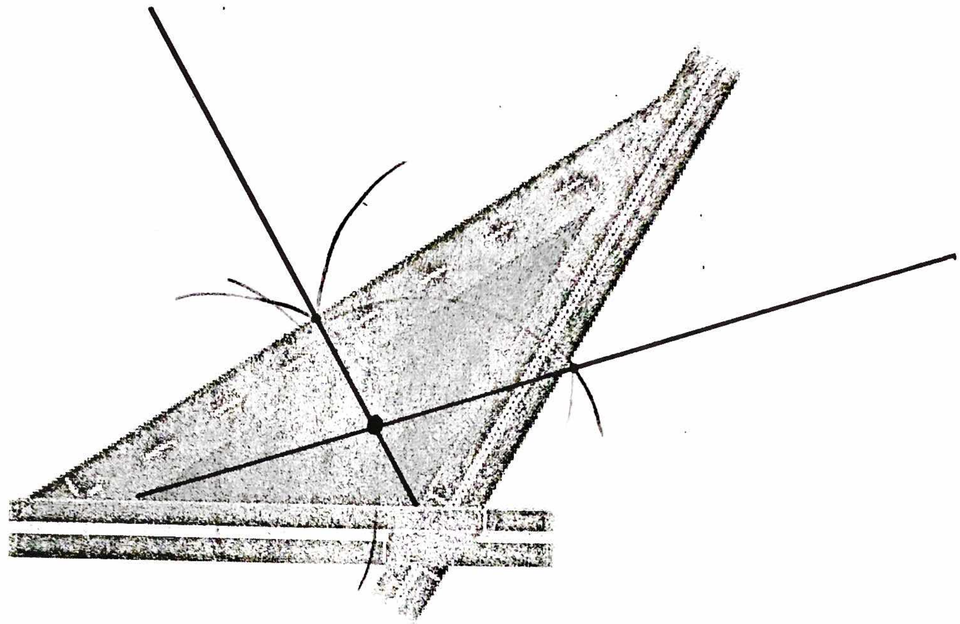
Examples:

4. A city wants to place a lamppost on the boulevard shown so that the lamppost of the same distance from all three streets. Should the location of the lamppost be at the *circumcenter* or the *incenter* of the triangle boulevard? Explain.

   → sides of Δ

   $\angle$ bisectors

5. Now, sketch where the lamppost should be!



6. Three snack carts sell hot pretzels from points A, B, and E. What is the location of the pretzel distributor if it is equidistant from the three carts? *Should it be at the circumcenter or the incenter?* Sketch the triangle and show the location.